Review: Product/Quotient Rule - 10/24/16

1 Product Rule

(2:25)

Product Rule: $\frac{d}{dx}f(x)g(x) = f'(x)g(x) + g'(x)f(x)$.

Example 1.0.1 *Let* $k(x) = (x^2+4)(x^3-7x+3)$. *Let* $f(x) = x^2+4$ *and* $g(x) = x^3-7x+3$. *Then* $k'(x) = f'(x)g(x) + g'(x)f(x) = (2x)(x^3-7x+3) + (3x^2-7)(x^2+4)$.

Example 1.0.2 *Let* $k(x) = x(7-\sqrt{x})$. *Let* $f(x) = x$ *and* $g(x) = 7-\sqrt{x}$. *Then* $f'(x) = 1$ *and* $g'(x) = -\frac{1}{2}x^{-\frac{1}{2}}$. *Then* $k'(x) = 7-\sqrt{x} + (-\frac{1}{2}x^{-\frac{1}{2}})x$.

Example 1.0.3 *Let* $k(x) = (3+e^x)\frac{1}{x^2}$. *We can rewrite this as* $k(x) = (3+e^x)x^{-2}$. *Let* $f(x) = 3+e^x$ *and* $g(x) = x^{-2}$. *Then* $f'(x) = e^x$ *and* $g'(x) = -2x^{-3}$, *so* $k'(x) = (e^x)(x^{-2}) + (-2x^{-3})(3+e^x) = \frac{xe^x}{x^3} - \frac{6+2e^x}{x^3} = \frac{-6+(x-2)e^x}{x^3}$.

2 Quotient Rule

Quotient Rule: $\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x)-g'(x)f(x)}{[g(x)]^2}$.

Example 2.0.4 *Let* $k(x) = \frac{3x+9}{2x}$. *Let* $f(x) = 3x+9$ *and* $g(x) = 2x$. *Then* $f'(x) = 3$ *and* $g'(x) = 2$, *so* $k'(x) = \frac{3(2x)-2(3x+9)}{4x^2} = \frac{-18}{4x^2}$.

Example 2.0.5 *Let* $k(x) = \frac{x^2}{2-x}$. *Let* $f(x) = x^2$ *and* $g(x) = 2-x$. *Then* $f'(x) = 2x$ *and* $g'(x) = -1$, *so* $k'(x) = \frac{2x(2-x)-(-1)(x^2)}{(2-x)^2} = \frac{4x-2x^2+x^2}{(2-x)^2} = \frac{4x-x^2}{(2-x)^2}$.

Example 2.0.6 *Let* $k(x) = \frac{(3+e^x)}{x^2}$. *Let* $f(x) = 3+e^x$ *and* $g(x) = x^2$. *Then* $f'(x) = e^x$ *and* $g'(x) = 2x$, *so* $k'(x) = \frac{e^x(x^2)-(3+e^x)(2x)}{x^4} = \frac{-6+(x-2)e^x}{x^3}$.

3 Higher Order Derivatives

Definition 3.0.7 *The derivative of the derivative of* f *is called the* ***second derivative*** *of* f, *and is denoted* $f''(x)$, *or* $\frac{d^2}{dx^2}f(x)$.

Definition 3.0.8 *The derivative of the second derivative is called the* ***third derivative*** *of* f *and is denoted* $f'''(x)$ *or* $\frac{d^3}{dx^3}f(x)$.

Definition 3.0.9 *In general, the* n*th derivative of* f *is denoted* $f^{(n)}$ *and is obtained by taking the derivative* n *times.*

Example 3.0.10 *What is* $\frac{d^2}{dx^2}x^3$*? It is* $\frac{d}{dx}3x^2 = 6x$.

Example 3.0.11 *What is* $\frac{d^3}{dx^3}e^x$*? It is* $\frac{d^2}{dx^2}e^x = \frac{d}{dx}e^x = e^x$*. Let* $f(x) = e^x$*. What is* $f^{(n)}(x)$*? It is still* e^x*!*

Example 3.0.12 *The first derivative of distance is velocity. The second derivative measures the change in velocity, so the second derivative of distance is acceleration.*

Practice Problems

1. Let $f(x) = (x^3 - 4)(\sqrt{x})$. What is $f'(x)$?
2. Let $f(x) = (x^3 - 4)(\sqrt{x})$. What is $f''(x)$?
3. Let $g(x) = \frac{x^3+8}{x-1}$. What is $g'(x)$?
4. Let $h(x) = \frac{(x+3)e^x}{x}$. What is $h'(x)$?

Solutions

1. Using the product rule, we get $f'(x) = (3x^2)(\sqrt{x}) + \left(\frac{1}{2\sqrt{x}}\right)(x^3 - 4) = \frac{(3x^2\sqrt{x})(2\sqrt{x})+x^3-4}{2\sqrt{x}} = \frac{7x^3-4}{2\sqrt{x}}$.
2. Using the quotient rule, we take the derivative of $f'(x)$, which we got from above. We get
$$f''(x) = \frac{(21x^2)(2\sqrt{x}) - (x^{-1/2})(7x^3 - 4)}{4x} = \frac{\frac{(21x^2)(2\sqrt{x})(\sqrt{x})-(7x^3-4)}{\sqrt{x}}}{4x} = \frac{35x^3 + 4}{4x\sqrt{x}}.$$
3. The quotient rule gives us $g'(x) = \frac{(3x^2)(x-1)-(1)(x^3+8)}{(x-1)^2} = \frac{2x^3-3x^2-8}{(x-1)^2}$.
4. We're going to need both the product rule and the quotient rule here: the product rule will give us the derivative of the numerator. Then $h'(x) = \frac{[(1)(e^x)+(e^x)(x+3)]x-1((x+3)e^x)}{x^2} = \frac{(x^2+4x)e^x-(x+3)e^x}{x^2} = \frac{(x^2+3x-3)e^x}{x^2}$.